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## Intra-Sector and Inter-Sector Competition

Marco DI CINTIO  
Emanuele GRASSI

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**Dipartimento di Scienze dell'Economia**

Indirizzo mail: [direzione.dipeconomia@unisalento.it](mailto:direzione.dipeconomia@unisalento.it)

Ecotekne - via Monteroni

73100 Lecce

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# Intra-Sector and Inter-Sector Competition in a Model of Growth

Marco Di Cintio  
Emanuele Grassi

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## Abstract

The role of patents is threefold: first, they are important to state the property rights of an invention; second, they are necessary to secure financing for starting a new venture; third, they are fundamental to recoup R&D investments. The main difficulty in preventing unauthorized use of an innovation is in the establishment of ranges and contexts of patents applicability. Noting the imperfections of the patent legal system, we are in a position to consider an economy with two levels of competition under different market structures: the inter-sector monopolistic competition and the intra-sector Cournot oligopoly. The explicit consideration of strategic interactions in a model of endogenous growth produces interesting results. Considering the sectorial market share as the indicator of patent system enforcement, we find that growth takes place, if and only if, there are some property rights of private knowledge produced by R&D activities. In turn, the patent system translates into a low degree of competition among firms. Its influence on the growth rate goes in a single unambiguous direction. As competition rises, few resources are available for R&D, so the growth rate goes down.

**Corresponding author:** marco.dicintio@unisalento.it

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## Introduction

The aim of this paper is to investigate the relationship between the growth rate and the intensity of market competition when monopolistic and oligopolistic

competition coexist in a model of expanding variety of products. The inter-sector monopolistic competition is more or less intense on the basis of the degree of substitutability among differentiated goods, while the degree of intra-sector competition depends on the number of active firms in each sector.

Remarkable contributions on endogenous growth theory are focused either on oligopoly or monopolistic competition. On the one hand, Romer (1990), Grossman-Helpman (1991) and Aghion-Howitt (1992) propose different approaches based on monopolistic competition to generate an endogenous process of knowledge acquisition, where they rely on the assumption that a large number of firms results in a negligible effect of individual choices on the aggregate price index. On the other hand, the difficulty of defining a balanced growth rate under differentiated oligopoly limits the scope of the literature under this market structure. However, the frequent adoption of the Dixit-Stiglitz (1977) aggregation method in models of growth (under monopolistic competition) may be well explained through its many attractive proprieties. First, the CES formulation of the utility function implies fair properties of the aggregate demand functions, i.e. a tractable analytical form. Second, a single (constant) parameter characterizes the degree of product differentiation (which is itself related to the “love for variety”, the degree of substitutability and the market power), facilitating the analysis between firms’ market power and the growth rate. The last property is the symmetry between old and new varieties, which removes product obsolescence and, as a consequence, excludes improvements in quality.

However, many economists have abandoned the hypothesis of monopolistic competition in order to introduce oligopolistic markets and to study the effects of strategic interaction on the growth rate. Remarkable contributions are those by Vencatachellum (1998), Peretto (1999) and Cellini (2000). Anyway also in the presence of strategic interaction, many papers usually rely on the assumption that a large number of firms results in a negligible effect (of individual choices) on the aggregate price index, even though this is acceptable only in a world of monopolistic competition<sup>1</sup>.

The literature typically conceives the two market structures as separate or unconnected and, sometimes, the distinction between oligopoly with differentiated goods and monopolistic competition is also unclear. Often, the two terms are used with a vague sense of imperfect competition: while the oligopoly describes few firms competing with or without free entry, the monopolistic competition refers to numerous firms and free entry<sup>2</sup>. By contrast, we study a framework where monopolistic and oligopolistic competition coexist at different levels. In particular, our aim is twofold: on the one hand, we propose a different approach where two market structures simultaneously coexist in a growth model; on the other hand, we study the influence of the degree of competition on the growth rate when strategic interaction really plays a role.

<sup>1</sup>See Yang-Heijdra (1993) and D’Aspremont et al. (1996).

<sup>2</sup>Following as example, Hart (1985) or Wolinsky (1986), the four standard properties of monopolistic competition are: (1) there are many firms producing differentiated commodities; (2) each firm is negligible; (3) free entry results in zero-profit of active firms; (4) the equilibrium price exceeds the marginal cost.

Our model is based on three simple ingredients. The first is related to the two dimensions of competition: the inter-sector monopolistic competition between differentiated products, and the Cournot oligopoly at the intra-sector level. The second is the traditional R&D technology à la Grossman-Helpman. The third is the assumption that the R&D output is of public domain. Because of the imperfections in the patent system, property rights may be difficult to define, so inventors are unable to exclude others from freely use their innovative ideas. The model explains clearly the relationship between the degree of market competition and the endogenous growth path. Sustained innovations are possible if, and only if, some *intellectual* property rights prevent the free use of an invention; otherwise, market tends to be highly competitive. In this case, few resources are available for R&D activity and the growth rate falls. By contrast when no firm has direct competitors, the state of knowledge moves forward because the private incentives for further research are maintained.

The discussion is organized as follows. The description of preferences is presented in section 1, while in section 2 we analyze the production side. Sections 3 and 4 describe the structure of R&D activities and the dynamic equilibrium. The last section concludes.

## 1 Preferences

Consider an economy with  $\bar{L}$  identical households and differentiated goods produced in  $N_m$  varieties,  $[x_i]_{i=1}^{N_m}$ . Preferences are identical for all consumers. Households maximize the lifetime utility:

$$U(t_0) = \int_{t_0}^{\infty} e^{-\rho(t-t_0)} \ln u(t) dt \quad (1)$$

subject to the intertemporal budget constraint, such that the present discounted value of expenditure cannot be greater than the present discounted value of lifetime labour income, plus initial wealth:

$$\int_{t_0}^{\infty} R(t)Y(t)dt \leq A(t_0) + \int_{t_0}^{\infty} R(t)w(t)dt \quad (2)$$

where  $\rho > 0$  is the individual discount rate,  $R(t) = e^{-\int_{t_0}^t r(s)ds}$  is the cumulative discount factor,  $Y$  is nominal per capita expenditure, and  $A$  is the initial wealth. The household takes the path of wages and the interest rate as given. Throughout the analysis, the wage is the numéraire.

We assume that there is a large number of varieties, all of which enter symmetrically into the instantaneous utility function  $u(t)$ , which we assume to be of the Dixit-Stiglitz type<sup>3</sup>:

$$u = \left( \sum_{i=1}^{N_m} x_i^\beta \right)^{\frac{1}{\beta}} \quad (3)$$

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<sup>3</sup>In the rest of the paper the time variable,  $t$ , is suppressed.

where  $x_i$  is the consumption of each variety and  $0 < \beta < 1$ . As it is well known, this specification has proved to be the most tractable when product differentiation is the main concern<sup>4</sup>. Over time, innovation can expand this subset, and  $N_m(t)$  is the number of varieties at time  $t$ . This utility function implies constant elasticity of substitution between any couple of varieties:

$$\sigma = \frac{1}{1 - \beta} > 1 \quad (4)$$

The solution of this problem can be derived in two stages. From the Euler equation, we first obtain the optimal dynamic expenditure path:

$$\frac{\dot{Y}}{Y} = r - \rho \quad (5)$$

which also defines optimal saving behavior. Then, by taking the time-path of expenditure as given, we solve the static household maximization problem for any  $t$ , i.e. the maximization of  $u$  subject to  $Y = \sum_{i=1}^{N_m} p_i x_i$ .

The  $h$ -th household's demand function for the  $i$ -th variety (where  $i \in [1, N_m]$ ) is

$$x_i^h(p_i) = \frac{Y}{q} \left( \frac{p_i}{q} \right)^{-\sigma} \quad (6)$$

where  $p_i$  is the price of the  $i$ -th brand, and  $q$  is the 'dual' price index:

$$q = \left[ \sum_{i=1}^{N_m} p_i^{1-\sigma} \right]^{\frac{1}{1-\sigma}} \quad (7)$$

Aggregating over  $\bar{L}$  identical consumers, we obtain the demand schedule faced by firms producing the  $i$ -th brand:

$$x_i(p_i) = \bar{L} \frac{Y}{q} \left( \frac{p_i}{q} \right)^{-\sigma} \quad (8)$$

Equation (8) is used in the analysis of firms' price-setting behavior. Since we are interested in quantity competition between firms, we consider the corresponding inverse demand function, along the lines suggested by Spence (1976):

$$p_i(x_i) = \bar{L} Y \frac{x_i^{\beta-1}}{Q^\beta} \quad (9)$$

where  $p_i$  is the price of the  $i$ -th variety,  $x_i$  is the aggregate production of the  $i$ -th sector, and  $Q$  is the industry quantity index given by:

$$Q = \left[ \sum_{i=1}^{N_m} x_i^\beta \right]^{\frac{1}{\beta}} \quad (10)$$

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<sup>4</sup>The love for variety could alternatively be modeled in a slightly different framework, by extending preferences over a continuous product space and assuming that at any given moment in time only a subset of potential varieties are available (Grossman and Helpman, 1989; Krugman, 1980).

Notice the immediate interpretation of  $\beta$  in terms of both market structure and preferences. As  $\beta \rightarrow 0$ , the degree of substitution between any couple of varieties reaches the minimum level (i.e.  $\sigma \rightarrow 1$ ) and varieties of different sectors become highly differentiated. As  $\beta \rightarrow 1$ , we obtain a set-up with an homogeneous product, the degree of substitutability becomes infinite (i.e.  $\sigma \rightarrow \infty$ ) and any brand is perfectly substitutable with any others of the remaining  $N_m - 1$  sectors. Clearly, the demand function given in (8) or (9) encompasses both traditional formulations of oligopoly with a homogeneous good, and the standard monopolistic competition.

## 2 Technology

On the production side, firms undertake two activities. First, they produce the existing varieties; second, they can divert resources to investment in R&D in order to create new designs.

While it is generally assumed that each variety is produced by a single firm, in what follows we will assume that each variety will be manufactured by  $N$  competing firms. This assumption can be justified in different ways. The innovative brand may not be patentable because its inventor has difficulties to prevent unauthorized use of its ideas. Alternatively, one may think at this kind of innovation as a new combination of existing knowledge. In the latter interpretation, the new product may indeed look new to consumers, but, being not really original, it is not patentable. Another way to justify our assumption is that, especially in the case of trade openness, similar varieties could exhibit many overlapping characteristics<sup>5</sup>, and a (nearly) identical brand is produced by many firms. Finally, we recall that Grossman and Helpman (1991) exclude any incentive to imitation on the basis that an intra-sector price competition would immediately lead profits to zero, so that the copier would not be able to recoup the positive cost of imitation. Their argument is clearly based on the idea that firms compete under a Bertrand fashion. But if we imagine Cournot competition, the scope for imitation may indeed arise. If the intra-sector competition is consistent with a positive mark-up over marginal costs, the imitation costs can be covered and firms could find it profitable to produce the same (homogeneous) good.

Since there are  $N_m$  varieties, each of them produced by  $N$  firms, each firm simultaneously faces two different competitive environments. Horizontally, at the inter-sector level each firm competes with other firms producing an imperfect substitute of its own product. Also, it competes with other firms producing a homogeneous product at the intra-sector level<sup>6</sup>. Therefore, there is an inter-sector competition (i.e. between different varieties) of the standard monopolistic

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<sup>5</sup>The case of the automobile sector provides clear examples in this respect.

<sup>6</sup>Notice that also in Grossman and Helpman (1991) there is a schematic discussion of possible forms of intra-sector competition. In particular they suggest that the research labs could be involved in quality improvements of existing varieties, so that intra-sector competition may turn to vertical product differentiation.

type, and an intra-sector competition (within the same variety). As suggested above, we assume that the latter is in quantities, so that the market for each variety can be thought as a traditional Cournot oligopoly.

The  $j$ -th firm ( $j \in [1, N]$ ) operating in the  $i$ -th sector, is mono-product. Each good can be produced through labour according to the linear technology:

$$z_{ij}(L_{ij}) = L_{ij} \quad (11)$$

where  $L_{ij}$  is the amount of labour employed in the  $i$ -th sector by the  $j$ -th firm, and  $z_{ij}$  is the firm's output. Hence, for the  $j$ -th firm, the cost function is  $C(z_{ij}) = z_{ij}$  (remember that the wage,  $w$ , is the numéraire). Obviously, the aggregate production for the  $i$ -th sector is:

$$x_i = \left( \sum_{j=1}^N z_{ij} \right) \quad (12)$$

Therefore, the number of workers employed in the  $i$ -th sector is given by:

$$L_i = \left( \sum_{j=1}^N z_{ij} \right) = x_i \quad (13)$$

while the total amount of workers employed in production is:

$$L_X = \left[ \sum_{i=1}^{N_m} \left( \sum_{j=1}^N L_{ij} \right) \right] \quad (14)$$

Each firm chooses the level of production in order to maximize profits:

$$\pi_{ij} = p_i(x_i)z_{ij} - z_{ij} \quad (15)$$

Notice that, given the large number of existing varieties, each firm perceives the industry quantity index as given. In turn, this implies that the negligibility assumption holds: each firm considers the change in its own level of production,  $z_{ij}$ , as irrelevant with respect to the industry aggregate production index,  $Q$ . Therefore it is the negligibility assumption that allows for the inter-sector monopolistic competition. On the contrary at the intra-sector perspective, competition is à la Cournot.

Substituting (9) into (15), and using (12), we can rewrite profits in terms of individual quantity:

$$\pi_{ij} = \bar{L}Y \frac{\left[ \sum_{j=1}^N z_{ij} \right]^{\beta-1} z_{ij}}{Q^\beta} - z_{ij} \quad (16)$$



Table 1: Equilibrium outcomes

Homogeneous product: $\beta \rightarrow 1$			
<i>inter-sector perfect competition</i>			
	$p$	$x$	$\pi$
	marginal cost	$\frac{LY}{N_m}$	0

The first order condition under Cournot conjectures for any given level of  $z_{hk}$ ,  $h \neq i$  and  $k \neq j$ , is

$$\frac{\partial \pi_{ij}}{\partial z_{ij}} = 0 \iff \bar{L} \frac{Y}{Q^\beta} \left[ (\beta - 1) \left( \sum_{j=1}^N z_{ij} \right)^{\beta-2} z_{ij} + \left( \sum_{j=1}^N z_{ij} \right)^{\beta-1} \right] - 1 = 0 \quad (17)$$

Under symmetry  $z_{ij} = z \forall i, j$ , the Nash equilibrium is:

$$z^* = \bar{L} Y \frac{(\beta - 1 + N)}{N^2 N_m}. \quad (1)$$

From (12), the aggregate production of each sector is:

$$x^* = \bar{L} Y \frac{(\beta - 1 + N)}{N N_m}, \quad (2)$$

and the related market price is given by (9)

$$p^* = \frac{N}{\beta - 1 + N}. \quad (3)$$

The resulting level of profits at the equilibrium is:

$$\pi^* = \bar{L} Y \frac{1 - \beta}{N^2 N_m} \quad (21)$$

Notice that the optimal quantity produced by any firm is inversely proportional to the number of existing varieties,  $N_m$ . The same holds for profits, while the price level is independent of  $N_m$ . Notice, also, the influence of the degree of substitutability. For a low level of  $\beta$ , inter-sector competition is less fierce because of the low interdependence among sectors. Table 1 summarizes the equilibrium outcome under these two extreme configurations of the inter-sector competition.

At the intra-sector level, a simple indicator of the degree of competition is given by the number of active firms in the sector. In this respect, on the one hand a large number (i.e.  $N \rightarrow \infty$ ) means that no limits to imitation exist; on the other hand, this implies a negligible market share for each firm of the sector (i.e.  $\frac{z}{x} = \frac{1}{N} = s \rightarrow 0$ ). In this case, the intra-sector competition resembles perfect competition: prices equal marginal costs and profits are driven to zero. On the

Table 2: Equilibrium outcomes

Free entry: $s \rightarrow 0$		
<i>intra-sector perfect competition</i>		
$p$	$x$	$\pi$
marginal cost	$LY \frac{1}{N_m}$	0

contrary, when a strict patent system prevents imitation and unauthorized entry into the sector, only a single firm supplies the entire sector ( $s = 1$ ), i.e. this firm behaves like a monopolist<sup>7</sup>. Table 2 summarizes the extreme configurations of intra-sector competition.

It must be stressed that the market share  $s$  can be interpreted in two different ways. It is an index of the degree of competition of market structure, but it can also be seen as an indicator of the degree of enforcement of patent law. In this respect, the extreme values,  $s \rightarrow 0$  and  $s = 1$ , arise under perfect competition (absence of patents) and monopoly power (perfect patents), respectively. For intermediate values of  $s$ , we have some degree of strategic interaction: the higher the value of  $s$ , the lower the degree of competition and the higher the level of patentability.

Finally, we recall that if the intra-sector competition were à la Bertrand, we would have the competitive price (because of homogeneity), independently of the properties of the inter-sector competition.

### 3 Research & Development

Following Grossman and Helpman (1991) and Lucas (1988), we assume that the production of new varieties takes place according to the innovation function:

$$\frac{\partial N_m}{\partial t} = \dot{N}_m = \frac{1}{a} L_R k(t) \quad (22)$$

where  $a$  is a positive parameter,  $L_R$  is the number of workers employed in R&D and  $k(t)$  is the stock of knowledge at time  $t$ . Equation (22) is the most common formulation of R&D technology in the endogenous growth literature: it shows a positive relationship between the development of new varieties and the stock of available knowledge at each moment in time. Since the number of varieties changes over time, the stock of knowledge depends, in a proportional way, on the number of existing varieties. This can be justified in terms of learning by doing: each innovation, by increasing the level of knowledge, makes R&D more productive.

The simplest function linking the stock of knowledge to the number of varieties is the linear one:

$$k(t) = N_m \quad (23)$$

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<sup>7</sup>This latter situation collapses to that described by the Grossman and Helpman model, where the intra-sector competition is absent.

and, on the basis of the R&D technology, the cost of the creation of a new variety is:

$$I_v(t) = \frac{a}{k(t)} = \frac{a}{N_m}. \quad (24)$$

Therefore, making use of (23), equation (22) determines the endogenous growth rate:

$$g = \frac{\dot{N}_m}{N_m} = \hat{N}_m = \frac{1}{a}L_R. \quad (25)$$

Assuming free entry in R&D activity, the present value of profits for any variety discovered at time  $t$  must be equal to its cost of creation:

$$V(t_0) = \int_{t_0}^{\infty} R(t)\pi(t)dt = \frac{a}{N_m}. \quad (26)$$

At each moment in time the Fisher equation must hold: the current profit plus the rate of capital gain must be equal to the value of profitable capital investment:

$$\pi_t + \dot{V}_t = rV_t. \quad (27)$$

Suppressing the time notation, and expressing (27) in proportional terms, we have:

$$\frac{\pi}{V} + \frac{\dot{V}}{V} = r. \quad (28)$$

The rate of profit is given by  $\frac{\pi}{V} = \bar{L}Y \frac{(1-\beta)}{aN^2}$ , while the percentage change in the present value of profits is  $\hat{V} = \frac{\dot{V}}{V} = -\hat{N}_m = -g$ , so that the Fisher equation can be rewritten as:

$$r = \bar{L}Y \frac{(1-\beta)}{aN^2} - g. \quad (4)$$

By using the labour market clearing condition, the amount of available labour is allocated between the two activities:  $L_X$  for production and  $L_R$  for R&D. If the supply of labour is fixed at the level  $\bar{L}$ , we have:

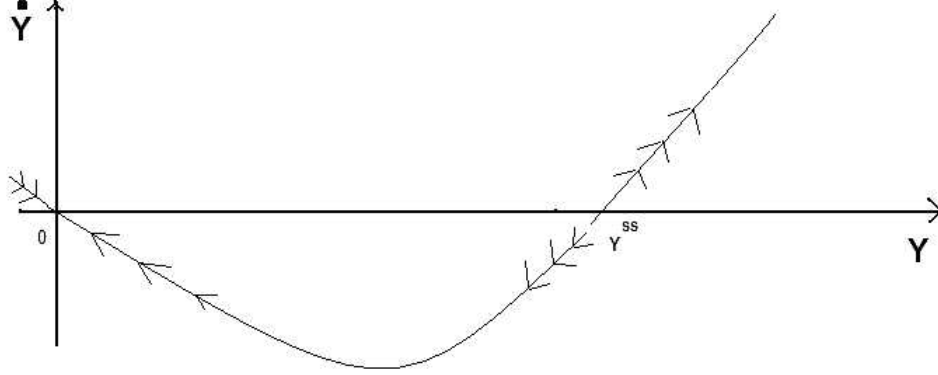
$$\bar{L} = L_R + L_X \quad (30)$$

Assuming full employment, the constraint on labour resources must always be satisfied. From (25) together with (13), (14) and (19), equation (30) can be rewritten in terms of the growth rate:

$$g = \frac{\bar{L}}{a} \left( 1 - Y \frac{(\beta - 1 + N)}{N} \right) \quad (31)$$

The higher the rate of innovation is, the greater the employment in R&D is and the lower the number of workers left for manufacturing. Therefore, over time, since new varieties are produced through the residual workers not employed in production, the aggregate production of each sector decreases at the rate  $g$ , and the number of available varieties increases at the same rate.

Figure 1: Phaseline.JPG



## 4 Dynamics

The general equilibrium is described by equations (5), (29) and (31). By substituting (31) into (29) for  $g$  we get

$$r = \frac{\bar{L}}{a} [Y (s^2(1 - \beta) - s(1 - \beta) + 1) - 1] \quad (32)$$

which in turn can be substituted for  $r$  into (5) in order to obtain the following dynamic equation in  $Y$  (where we have used the definition of  $s$ ):

$$\dot{Y} = Y^2 \left( \frac{\bar{L}}{a} [s^2(1 - \beta) - s(1 - \beta) + 1] \right) - Y \left( \frac{\bar{L} + a\rho}{a} \right). \quad (33)$$

This Bernoullian equation has two steady state solutions. The graph of  $\dot{Y}$  (figure 1) cuts the horizontal axis twice, at the origin and at  $Y^{ss}$ . The first solution,  $Y = 0$ , is stable. The second is unstable and is given by:

$$Y^{ss} = \frac{a\rho + \bar{L}}{\bar{L}} \frac{1}{s^2(1 - \beta) - s(1 - \beta) + 1}. \quad (34)$$

The qualitative properties of equation (33) can be described through a phase line.

For all values of  $Y$  within the interval  $]0, Y^{ss}[$ , expenditure must be decreasing, indicating that  $\dot{Y} < 0$ . For values of  $Y > Y^{ss}$  the opposite holds,  $\dot{Y} > 0$  and expenditure increases.

While stable, the first solution ( $Y = 0$ ) is economically meaningless. If in the long run the aggregate expenditure approaches zero, the rate of innovation reaches its maximum value; in this situation the entire supply of labour is employed in R&D and there is no production activity. However, in this case, while the number of products would be growing continuously at the positive rate

$g = \frac{\bar{L}}{a}$ , expenditure and profits would approach zero and the arbitrage condition would be violated: the present value of profit would be lower than the positive entry cost. By contrast, the second solution, though unstable, is economically meaningful. Therefore, we must impose stability by assuming that starting from any initial value  $Y$ , being a non-predetermined variable, it jumps instantaneously to  $Y^{SS}$ . In the steady state, constant household's expenditure must involve a constant interest rate which exactly matches the subjective discount rate ( $r = \rho$ )<sup>9</sup>.

We now substitute the steady state equilibrium values of  $Y^{SS}$  and  $r$  into the Fisher equation, in order to obtain the steady state solution for the growth rate<sup>10</sup>:

$$g^{SS} = \frac{\rho a [s(1 - \beta) - 1] + \bar{L}(1 - \beta)s^2}{s^2(1 - \beta) - s(1 - \beta) + 1} \frac{1}{a}. \quad (35)$$

In order to analyze the properties of the steady state solution, it is useful to see it explicitly as the (simultaneous) solution (for  $Y$  and  $g$ ) of the labour market clearing condition (31) and the free entry condition in R&D given by the Fisher equation (29), the latter evaluated at  $r = \rho$ <sup>11</sup>

$$g = \bar{L}Y \frac{(1 - \beta)}{a} s^2 - \rho \quad (36)$$

$$g = \frac{1}{a} \bar{L} (1 + Y [s(1 - \beta) - 1]). \quad (5)$$

These two linear equations are represented in Figure 2.

Equation (36) is positively sloped in the  $(Y, g)$  plane, while equation (37) is negatively sloped since  $0 < s \leq 1$  and  $\beta < 1$ . The intersection between the two equations gives us the same steady state equilibrium values (34) and (35) for  $Y$  and  $g$ . The intersection point is one where the splitting of labour resources between production and R&D, remains constant over time. In this respect, the rate of product development exactly matches the rate of decline of entry cost, and innovation occurs at the same constant rate  $g^{SS}$ .

Now, we are in a position to evaluate the relationship between the degree of competition of market structure (captured by the market share  $s$ ) and the growth rate  $g^{SS}$ . Considering the position of the two equations, the higher  $s$  is (i.e. the lower the degree of competition is), the greater the (positive) slope of the first equation (36), and the smaller the (negative) slope in (37). The total

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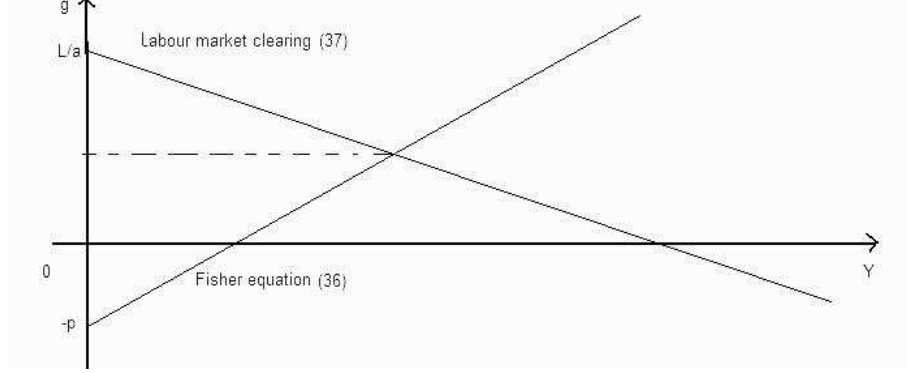
<sup>8</sup>Even though it would seem questionable in Industrial Organization literature, it is common in the endogenous growth models

<sup>9</sup>In this set-up, the assumption that  $Y$  jumps to its steady state equilibrium (other than  $Y = 0$ ) is the equivalent of Grossman and Helpman hypothesis that nominal expenditure is normalized and constant to one.

<sup>10</sup>Notice that in the admitted range of  $s$  and  $\beta$ , the denominator of (35) is positive. Moreover, we refer to specifications of  $\bar{L}$ ,  $a$  and  $\rho$  such that the growth rate to be meaningful. Violating these specifications, economy immediately jumps in a stationary state without innovation (also see: Grossman and Helpman (1991), Romer (1990) for insufficient endowment).

<sup>11</sup>Obviously, for  $r = \rho$  expenditure is constant over time; in this respect this level of the interest rate is the sole compatible with a constant expenditure.

Figure 2: Solution



effect is an higher growth rate. On the contrary, when the degree of competition is high (i.e. the level for  $s$  is low), the resulting growth rate is lower.

The interpretation of this relationship is straightforward. Suppose the lowest level for  $s$ , i.e.  $s \rightarrow 0$ , the firms' market share is negligible and the intra-sector market tends to be highly competitive. The equality between price and marginal cost implies the highest production level: more workers are employed in production, few resources are available for R&D activity and, obviously, the resulting growth rate falls. By contrast, when no firm has direct competitors, more workers are available for R&D activities, and so, the growth rate raises. Furthermore, a lower degree of interdependence among firms leads to a higher level of profits, giving more incentives to innovation activities.

The relationship between the growth rate and market share can also be analyzed by evaluating the derivative of  $g^{SS}$  in (35) with respect to  $s$ :

$$\frac{\partial g^{SS}}{\partial s} = \frac{[2 - s(1 - \beta)] [s(1 - \beta)] \left[ \rho + \frac{\bar{L}}{a} \right]}{[s^2(1 - \beta) - s(1 - \beta) + 1]^2} > 0.$$

Sustained innovations should be possible for  $s \neq 0$ . This means that a positive growth rate results if, and only if, some intellectual property rights prevent the free use of innovations.

Notice that when  $s = 1$ , there is only one firm per sector and this implies the traditional Grossman-Helpman outcome:

$$g_{G-H}^{SS} = \frac{\bar{L}}{a}(1 - \beta) - \rho\beta.$$

In their formulation,  $\beta$  is the parameter that plays a fundamental role with respect to the degree of competition in the market (captured by the degree of product differentiation). On the one hand, the lower the level of substitutability between varieties ( $\beta \rightarrow 0$ ), the higher the level of profits; thus the growth rate rises. On the other hand, when the degree of product differentiation is minimum

(i.e.  $\beta \rightarrow 1$ ), the profits are driven to zero because no firms have market power. In this set-up<sup>12</sup>, there is no incentive for product innovation because a new brand does not yield appropriate returns to the inventor in the form of a stream of monopoly profits.

## 5 Conclusion

In recent attempts to involve the strategic interaction in endogenous growth models an ambiguous relationship emerges between the growth rate and the degree of market competition. On the one hand, remarkable contributions on endogenous growth theory are focused on monopolistic competition. On the other hand, the difficulty of defining a balanced growth rate under differentiated oligopoly limits the scope of the economic literature under this market structure. By contrast, we study a framework where monopolistic and oligopolistic competition coexist at a different level, and growth takes place by expanding product variety.

Our model is based on three simple ingredients. The first is related to the two dimensions of competition: the inter-sector monopolistic competition between differentiated products, and the Cournot oligopoly at the intra-sector level. The second concerns the traditional R&D technology à la Grossman-Helpman. Third, because of the imperfections in the patent system, we assume that property rights may be difficult to define, so the inventors will be unable to exclude others from making free use of their innovative ideas.

The model explains clearly the relationship between the degree of competition and the endogenous growth rate. Sustained innovations are possible if, and only if, some intellectual property rights prevent the free use of an invention; otherwise, market tends to be highly competitive. In this case, few resources are available for R&D activity and the growth rate falls. By contrast when no firm has direct competitors, the state of knowledge moves forward because the private incentives for further research are maintained.

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<sup>12</sup>The same situation which occurs when no patents exist ( $s \rightarrow 0$ ).

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