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Asymmetric Information in Fads Models: Some extensions

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Outline

- Motivation:
  Ornstein-Uhlenbeck, Hitsuda Representation, Fads models, Asymmetric Information.
- Model:
  Continuos-time version for informed and uninformed agents.
- Results:
  Utility Maximization Problem.

$\Longrightarrow$ **Paolo Guasoni**: Asymmetric Information in Fads Model Finance and Stoch 10,159-177 (2006)

Abstract

This paper covers asymmetric information in financial markets from a micro perspective. Particularly, we aim to extend the asset pricing framework introduced by Guasoni (2006), which models price dynamics with both a martingale component, described by permanent shocks, and a stationary component, given by temporary shocks. First, we derive a generalization of this asset pricing model using two Ornstein-Uhlenbeck processes, then three and in the last case n Brownian motions, as well as include an Ornstein-Uhlenbeck process as the (n+1)th element. We find non-Markovian dynamics for the uniformed agents, which questions the validity of the efficient market hypothesis. Moreover, we contrast the positions of informed and uninformed agents. Thereby, the filtration for informed agents is larger and initially specified, whereas the filtration for uniformed agents is smaller and obtained from the Hitsuda representation (1968). For both agents, our study yields similar results as the findings of Guasoni, for the logarithmic utility maximization problem.

Set-up

- Introduce a $(\Omega, \mathcal{F}, P)$ on which are defined two indipendent Brownian motion $(W_t)_{t\in[0,+\infty)}$ and $(B_t)_{t\in[0,+\infty)}$.
- We denote by U the process Ornstein - Uhlenbeck obtained as the unique solution to the following stochastic differential equation:

$$dU_t = -\lambda U_t dt + dB_t; \tag{1}$$

- Set

$$Y_t = pW_t + qU_t \tag{2}$$

  where p,q > 0 and $p^2 + q^2 = 1$.
  Let's introduce two deterministic, Lebesgue measurable functions $\mu_t > 0$ e $\sigma_t > 0$ such that $\int_0^T \mu_t dt < +\infty$ and $\int_0^T \sigma_t^2 dt < +\infty$ for all $T > 0$.
- The price S of the risky asset is defined as the solution to the stochastic differential equation:

$$\frac{dS_t}{S_t} = \mu_t dt + \sigma_t dY_t \tag{3}$$

...Set-up

- which is

$$S_t = S_0 \exp\left[\int_0^t \left(\mu_s - \frac{\sigma_s^2}{2}\right) ds + \int_0^t \sigma_s dY_s\right]. \tag{4}$$

- introduce two filtrations $(\mathcal{F}_t^0)_{t\in[0,+\infty)}$ and $(\mathcal{F}_t^1)_{t\in[0,+\infty)}$, with $\mathcal{F}_t^0 \subset \mathcal{F}_t^1$, where $\mathcal{F}_t^0$ is generated by Y and $\mathcal{F}_t^1$ is generated by Brownian motions.

Extensions:

- Two processes O-U;
- Three processes O-U;
- n Brownian Motions and (n+1)th process O-U.

Asset Price Dynamics

For informed agents the asset price dynamics can be written as

$$\frac{dS_t}{S_t} = (\mu_t - \sigma_t q \lambda U_t)dt + \sigma_t dB_t^1 \tag{5}$$

where $B_t^1 = qB_t + pW_t$ is a Brownian motion.
Extensions (Two O-U):

$$\frac{dS_t}{S_t} = (\mu_t - \sigma_t q \lambda U_t - \sigma_t p \kappa V_t)\, dt + \sigma_t dB_t^1 \tag{6}$$

where $B_t^1 = qB_t + pW_t$ is a Brownian motion.

.. cont'd

- First task: estabilish a similar decomposition for S, or Y, from the viewpoint of the uninformed agents, in terms of some $\mathcal{F}^0$-Brownian motion B^0.
- Y fails the Markov property in its natural filtration.
- It isn't possibly expect a dynamics of the form

$$dY_t = \alpha\,(t, Y_t)\,dt + \beta\,(t, Y_t)\,dA^0_t. \tag{7}$$

Theorem

Let $Y_t = pW_t + qU_t$ and $(\mathcal{F}_t^0)_{t\in[0,+\infty)}$. Let $p, q, \lambda > 0$ and $p^2 + q^2 = 1$. Define:

$$\Gamma(s) = -\frac{1}{\lambda} \log\left[\cosh(\lambda ps) + p \sinh(\lambda ps)\right] \tag{8}$$

$$\gamma(s) = \Gamma'(s) = -p \tanh(\lambda ps + \operatorname{arctan} h(p)). \tag{9}$$

then the process

$$B_t^0 = Y_t + \int_0^t \left[\lambda(\gamma(s)+1)Y_s + \int_0^t \lambda^2(\gamma(s)+p^2)e^{\lambda(\Gamma(s)-\Gamma(u))Y_u du}\right] ds \tag{10}$$

is a $\mathcal{F}^0$ Brownian motion.

...cont'd

We can read the Theorem in the case of uninformed agent, and the asset price dynamics is

$$\frac{dS_t}{S_t} = (\mu_t - \sigma_t \nu_t)dt + \sigma_t dB_t^0 \tag{11}$$

where

$$\nu_t = -\lambda(\gamma(t)+1)Y_t - \lambda^2 \int_0^t (\gamma(s) + p^2) e^{\lambda(\Gamma(t)-\Gamma(u))} Y_u du \tag{12}$$

or

$$\nu_t = -\lambda \int_0^t e^{-\lambda(t-u)(1+\gamma(u))} dB_u^0 \tag{13}$$

Extension-Theorem (Two O-U)

Let $Y_t = pV_t + qU_t$ and $(\mathcal{F}_t^0)_{t\in[0,+\infty)}$. Let $p, q, \kappa, \lambda > 0$ and $p^2 + q^2 = 1$. Define:

$$\Gamma(s) = -\frac{1}{(\lambda+\kappa)} \log\left[a\cosh(a(\lambda+\kappa)s) + b\sinh(a(\lambda+\kappa)s)\right] \tag{14}$$

$$\gamma(s) = \Gamma'(s) = -a\tanh[a(\lambda+\kappa)s + \arctan h(b/a)]. \tag{15}$$

then the process

$$\begin{aligned} B_t^0 =& Y_t + \int_0^t \left\{ \int_0^s \left[(\lambda+\kappa) e^{(\lambda+\kappa)(\Gamma(s)-\Gamma(u))}(1+\gamma(u)) \right.\right. \\ & \left.\left. + \lambda\kappa \int_u^s \left(e^{(\lambda+\kappa)(\Gamma(s)-\Gamma(v))} \right) dv \right] dY_u \right\} ds \end{aligned} \tag{16}$$

is a $\mathcal{F}^0$- Brownian motion.

Extension Three O-U

- Let $Y_t = p_1 U_1 + p_2 U_2 + p_3 U_3$ and $(\mathcal{F}_t^0)_{t \in [0,+\infty)}$.
- Let $p_1, p_2, p_3, \lambda_1, \lambda_2, \lambda_3 > 0$.
- Y_t fails the Markov property in its natural filtration iff the following equality is not verified

$$\lambda_1 = \lambda_2 = \lambda_3.$$

We have to consider the "analogous" $\Gamma(s), \gamma(s), B_t^0$ like in the previous cases.

Extension n Brownian Motions and (n+1)th process O-U

- Let $Y_t = \sum_{j=1}^{n} p_j B_t^j + p_{n+1} U_t^{n+1}$ and $(\mathcal{F}_t^0)_{t\in[0,+\infty)}$.
- Let $\lambda_{n+1} > 0, p_{n+1} > 0$.
- Y_t fails the Markov property in its natural filtration iff one of the following equalities is not verified
  1) $\lambda_{n+1} = 0$;
  2) $\lambda_{n+1} > 0$ and $p_j = 0$ or $p_{n+1} = 0$.

We have to consider the "analogous" $\Gamma(s), \gamma(s), B_t^0$ like in the previous cases.

Set-up

- Assume that both informed and uninformed agents invest in the market model to maximize their expected utility from terminal wealth.
- The discounted portfolio value at time t of an agent starting with initial capital x and holding H_t shares of the asset S is given by:

$$X_t = x + (H \cdot S)_t \tag{17}$$

- Agents are costraint to use admissible strategies, which must be predictable in theri respective filtrations:
$A^i_x =$
$\left\{H : \mathcal{F}^i - predictable, S - integrable, x + (H \cdot S)_t > 0 \ a.s. \ \forall t \in [0, T]\right\}$

...cont'd

- We solve the problem of Utility maximization from terminal wealth:

$$\mathbf{max}\left\{\mathbf{E}[\mathbf{U}(\mathbf{X_T})] : \mathbf{H} \in \mathbf{A^i_x}\right\}$$

To write the portfolio value in exponential form, we denote by $\pi = \dfrac{HS}{X}$ the fraction of wealth in the risky asset, and observe that (17) can be written as

$$X_t = x + (H \cdot S)_t \;\Rightarrow X_t = x + \int_0^t H_s dS_s,$$

$$\pi = \frac{HS}{X} \;\Rightarrow H = \frac{\pi X}{S},$$

so

$$X_t = x + \int_0^t \frac{\pi_s X_s}{S_s}\, dS_s$$

$$dX_t = \frac{\pi_s X_t}{S_t}\, dS_t \;\Rightarrow \frac{dX_t}{X_t} = \pi_s \frac{dS_t}{S_t}.$$

...cont'd

And substituting the dynamics of the asset S_t

$$dX_t = X_t \left[\pi_t \mu_t dt + \pi_t \sigma_t dY_t\right].$$

where

$$X_t = x \exp\left[\int_0^t (\pi_s \mu_s - \frac{1}{2}\pi_s^2 \sigma_s^2) ds + \int_0^t \pi_s \sigma_s dY_s\right].$$

...cont'd

Bacause Y_t is not markovian, we have to consider B_t^0 and B_t^1 for the uniformed and informed agents:

- $dX_t = X_t \pi_t \left[\mu_t + \sigma_t \nu_t\right] dt + X_t \sigma_t \pi_t dB_t^0$
- $dX_t = X_t \pi_t \left[\mu_t - \lambda \sigma_t q U_t\right] dt + X_t \sigma_t \pi_t [p dW_t + q dB_t]$

Then

$$X_t = x \exp\left[\int_0^t (\pi_t \mu_t + \pi_t \sigma_t \nu_t - \frac{1}{2}\sigma_t^2 \pi_t^2) dt + \int_0^t \sigma_t \pi_t dB_t^0\right].$$

$$\begin{aligned} X_t =& x \exp\left[\int_0^t \left[\pi_t \left(\mu_t - \lambda \sigma_t q U_t - \kappa \sigma_t p V_t\right) - \frac{1}{2}\pi_t^2 \sigma_t^2\right] dt \right. \\ & \left. + \int_0^t \pi_t \sigma_t p dW_t + \int_0^t \pi_t \sigma_t q dB_t\right]. \end{aligned}$$

...cont'd

We consider the logarithmic utility maximization problem for both agents:

- $E(\log X_T) =$
  $\log x + E\left[\int_0^T (\pi_t\mu_t + \pi_t\sigma_t\nu_t - \frac{1}{2}\sigma_t^2\pi_t^2)dt\right] + E\left[\int_0^T \sigma_t\pi_t dB_t^0\right]$
- $E(\log X_T) = \log x + E\left[\int_0^T \left(\pi_t\mu_t - \pi_t\lambda\sigma_t qU_t - \frac{1}{2}\sigma_t^2\pi_t^2\right) dt\right] +$
  $E\left[\int_0^T \sigma_t\pi_t p dW_t\right] + E\left[\int_0^T \sigma_t\pi_t q dB_t\right]$

Theorem

The value function for u^i and the optimal strategies π^i for $i = 0, 1$ are

$$\pi_t^0 = \frac{\mu_t + \sigma_t \nu_t}{\sigma_t^2}, \qquad u^0(x) = \log x + \frac{1}{2} E\left[\int_0^T \left(\frac{\mu_t + \sigma_t \nu_t}{\sigma_t}\right)^2 dt\right].$$

$$\pi_t^1 = \frac{\mu_t - \lambda \sigma_t q U_t}{\sigma_t^2}, \qquad u^1(x) = \log x + \frac{1}{2} E\left[\int_0^T \left(\frac{\mu_t - \lambda \sigma_t q U_t}{\sigma_t}\right)^2 dt\right].$$

Then the following asymptotics, as $T \to +\infty$:

$$u^0(x) \sim \frac{1}{2} \int_0^T \left(\frac{\mu_t^2}{\sigma_t^2}\right) dt + \frac{\lambda}{4}(1-p)^2 T, \; u^1(x) \sim \frac{1}{2} \int_0^T \left(\frac{\mu_t^2}{\sigma_t^2}\right) dt + \frac{\lambda}{4}(1-p^2) T,$$

And the additional logarithmic utility of the informed agent is:

$$u^1(x) - u^0(x) \sim \frac{\lambda}{2} p(1-p) T.$$

Extension-Theorem (Two O-U)

The value function for u^i and the optimal strategies π^i for $i = 0, 1$ are

$$\pi_t^0 = \frac{\mu_t + \sigma_t \nu_t}{\sigma_t^2},$$

$$u^0(x) = \log x + \frac{1}{2} E\left[\int_0^T \left(\frac{\mu_t + \sigma_t \nu_t}{\sigma_t}\right)^2 dt\right];$$

$$\pi_t^1 = \frac{\mu_t - \lambda\sigma_t q U_t - \kappa\sigma_t p V_t}{\sigma_t^2},$$

$$u^1(x) = \log x + \frac{1}{2} E\left[\int_0^T \left(\frac{\mu_t - \lambda\sigma_t q U_t - \kappa\sigma_t p V_t}{\sigma_t}\right)^2 dt\right].$$

Then asymptotically, as $T \to +\infty$, we obtain that both utilities and the additional logarithmic utility of the informed agents diverge.

For both agents, in the case of n Brownian motions and (n+1)th O-U, our study yields similar results as the findings of Guasoni.

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